

Similarity Rules for Scaling Solar Sail Systems

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Future science missions will require solar sails on the order 10,000 m² (or larger). However, ground and flight demonstrations must be conducted at significantly smaller sizes (400 m² for ground demo) due to limitations of ground-based facilities and cost and availability of flight opportunities. For this reason, the ability to understand the process of scalability, as it applies to solar sail system models and test data, is crucial to the advancement of this technology. This report will address issues of scaling in solar sail systems, focusing on structural characteristics, by developing a set of similarity or similitude functions that will guide the scaling process. The primary goal of these similarity functions (process invariants) that collectively form a set of scaling rules or guidelines is to establish valid relationships between models and experiments that are performed at different orders of scale. In the near term, such an effort will help guide the size and properties of a flight validation sail that will need to be flown to accurately represent a large, mission-level sail.

I. Introduction

The NASA In-Space Propulsion Technologies (ISPT) Program is heading an effort to develop the capability of solar sails to perform a variety of new space observation and exploration missions. Solar sails can potentially provide low-cost propulsion and operate without the use of propellant, allowing access to non-Keplerian orbits through constant thrust. Solar sails consist of a gossamer structure, a membrane-based, large, lightweight structure. The primary objective of the sail is to convert solar pressure into thrust on a spacecraft. This solar pressure is extremely small, on the order of nine Newtons per square kilometer at one AU. Therefore, the fundamental challenge in solar sail design is to create a sail area of significant size, with small enough system mass to achieve reasonable accelerations of the spacecraft. A number of research activities are currently being conducted on solar sails, managed through the ISPT Program by NASA. These activities include a focus on developing advanced modeling and simulation techniques for solar sail systems, and in solar sail system development, ground demonstration and testing of a four-quadrant sail configuration. The objective of this work is to push the technology readiness level (TRL) of this propulsion system to a sufficient level to prepare for flight demonstration. Flight demonstration will then lead to a system ready to perform a specific science mission. The science missions envisioned will require sails on the order of 10,000 m² (or larger). However, ground demonstrations and flight demonstrations must be conducted at significantly smaller sizes, due to the limitations of ground-based facilities and cost and availability of flight opportunities. For this reason, a well understood process of scalability, as it applies to solar sail system models and test data, is crucial to the advancement of this technology. This paper will address issues of scaling in solar sail systems by developing a set of similarity or similitude functions that will guide the scaling process. The primary purpose of these similarity functions (process invariants) that collectively form a set of scaling rules or guidelines is to express valid relationships between models and experiments that are performed at different orders of scale. In the longer term, such an effort will help guide the size and properties of an experiment or test sail that would need to be flown to accurately represent a large, mission-level sail. This paper will merely review a preliminary development of this set of similarity rules, based on simple mechanics models for the sail system. Further work in this area will need to be conducted to completely achieve the stated goals.

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II. Background – Solar Sail Scaling

The ability to scale solar sail systems has been investigated by a number of researchers. For example, Holland et al. [1] observed a set of geometric scaling properties for inflatable structures while Greschik et al [2] have investigated a specific geometric scaling technique based on constant thickness. Issues are observed in a general sense by [3], and include geometric and dynamic scaling properties for solar sails and pragmatic issues in sail packing and deployment. A general procedure for developing similarity rules, and applications to a variety of examples are presented in [4], [5] and [6]. This report adds to the current literature an effort to enumerate the dimensionless parameters associated with the governing equations of a sail system, and to observe the degree to which experimental test results on a sail can be scaled.

III. Procedure

This paper presents the rules or criteria that should be considered in testing and analysis of solar sail system models. These rules, called similarity criteria, are derived directly from the governing equations of motion and boundary conditions of a sail system. The governing equations are cast into dimensionless form, and similarity criteria and characteristic parameters are defined from these equations. This analysis considers all of the similarity criteria that arise from the equations of motion considered. All phenomena that are included in the governing equations of motion employed here are covered by the similarity criteria.

The motivation for this effort is to provide guidelines for the process of designing scale models for testing and evaluation of an actual sail system. Due to a number of issues, the testing will be performed on test sails that are many times smaller than a mission-size or actual sail (for example, a mission sail may have a defining dimensional size of 100 m, while the test articles may be on the order of 10 m). However, it is desirable that the response of the test sail will accurately predict or characterize the response of the mission-size sail. Therefore, the primary purpose of this work is to define the criteria, known as similarity criteria, which can be used to guide the proper design of scaled sail models for analysis and testing. This work will also be used to give a basis for the degree of scaling that practically can be applied to the results of a given test article. This work will be used to demonstrate some basic key functional relations of sail behavior based on sail size. Finally, the work will provide some simple guidelines that can be used in optimal sail design and will provide the starting point for one approach to sensitivity analysis.

The remainder of this paper will proceed as follows. The similarity criteria that apply to the sail material first will be derived, followed by the criteria for the boom model. The sail material and boom models will then be combined through appropriate boundary conditions, resulting in a complete list of similarity criteria and corresponding characteristic terms. Once derived, the use of these similarity criteria will be discussed and demonstrated. Several examples will illustrate the use of similarity criteria in guiding the design of scaled models. An analysis of the extent of scaling that reasonably can be applied will be performed using the following process. In the potential situation that more similarity criteria exist than available design parameters, a true-scale model of the sail cannot be created. This test sail model that does not satisfy all the similarity criteria will be termed a sub-scaled test model. As the sub-scaled model is tested at increasing scale sizes (departing from nominal), its response on key characteristic behavior will diverge from that expected of the actual sail system. The degree of divergence that is permissible on a specific parameter can be then used to define the degree of scaling that could be deemed allowable. Finally, a discussion of future work with these scale models will be presented.

IV. Analytical Sail Models

A. Sail System Model

Consider as a sail model a thin-plate formulation that will allow a variable triaxial stress state to exist over the sail of a defined thickness. This model assumes an isotropic sail material of constant thickness, h , and constant pressure loading, q . This thin plate model is correct when bending stiffness is negligible with respect to extensional stiffness and the deflection is on the order of material thickness (Von Karmen plate or Foppl membrane equations). As a result, wrinkling is not considered. The equations of motion for this thin-plate model for the sail are provided by Timoshenko [7] and Mierovitch [8] as,

$$F_{xxx} + 2F_{xyy} + F_{yyy} = E((w_{xy})^2 - w_{xx}w_{yy}) \quad (1)$$

and

$$\frac{q}{h} + F_{yy}w_{xx} + F_{xx}w_{yy} - 2F_{xy}w_{xy} = \rho h w_{tt} \quad (2)$$

where Eq. 1 defines compatibility while Eq. 2 defines Newtonian equilibrium and F is the stress function, w the sail deflection, E the sail material modulus, q is the lateral sail loading, and ρ the sail density. The stress function is defined in terms of the in-plane loads as;

$$F_{yy} = N_x/h, \quad F_{xx} = N_y/h, \quad F_{xy} = -N_{xy}/h \quad (3)$$

with N_i the in-plane loading per length.

Using the techniques found in [4], the equations of motion for the thin-plate sail model can be written in dimensionless form:

$$\left(\frac{F_c}{x_c^4}\right)F_{xxxx} + 2\left(\frac{F_c}{x_c^2 y_c^2}\right)F_{xxyy} + \left(\frac{F_c}{y_c^4}\right)F_{yyyy} = E \left[\left(\frac{w_c}{x_c y_c}\right)^2 (w_{xy})^2 - \left(\frac{w_c^2}{x_c^2 y_c^2}\right) w_{xx} w_{yy} \right] \quad (4)$$

$$\frac{q}{h} + \left(\frac{F_c w_c}{x_c^2 y_c^2}\right)F_{yy} w_{xx} + \left(\frac{F_c w_c}{x_c^2 y_c^2}\right)F_{xx} w_{yy} - 2\left(\frac{F_c w_c}{x_c^2 y_c^2}\right)F_{xy} w_{xy} = \left(\frac{\rho h w_c}{t_c^2}\right)w_{tt} \quad (5)$$

where the transformation coefficients, F_c , x_c , y_c , w_c and t_c are defined as ratios of the corresponding quantities in the dimensional model (F , x , y , w and t) and in the prototype (F' , x' , y' , w' and t'); for example,

$$F_c = \frac{F'}{F}, \quad x_c = \frac{x'}{x}, \quad \text{etc}$$

and the dimensional variables from Eqs. 1-3 are now used to represent the dimensionless terms. Since these equations are now non-dimensional (expressed in terms of the dimensionless prototype), the coefficient terms are equivalent and determine the conditions for similitude. Equations 4-5 are first modified by dividing each through by an appropriate coefficient term resulting in equations that are now described with invariant coefficients, comprised of characteristic values, w_c , F_c and t_c describing the behavior of the scaled system and similarity criteria, x_c/y_c , that define the rules for scaling, yielding,

$$F_{xxxx} + 2P_1 F_{xxyy} + P_1^2 F_{yyyy} = (w_{xy})^2 - w_{xx} w_{yy} \quad (6)$$

$$1 + F_{yy} w_{xx} + F_{xx} w_{yy} - 2F_{xy} w_{xy} = w_{tt} \quad (7)$$

with

$$P_1 = \frac{x_c^2}{y_c^2}, \quad \text{geometric similarity criterion on sail,} \quad (8)$$

$$w_c = \sqrt[3]{\frac{q x_c^4}{h E P_1^2}}, \quad \text{characteristic sail displacement,} \quad (9)$$

$$F_c = \sqrt[3]{\frac{E q^2 x_c^8}{h^2 P_1^2}}, \quad \text{characteristic stress function and} \quad (10)$$

$$t_c = \sqrt{\frac{\rho h x_c^4}{q P_1}}, \quad \text{characteristic sail period.} \quad (11)$$

The geometric similarity criterion on the sail preserves the ratio of sail sizes and can be used to specify the size of a scaled sail accordingly,

$$\frac{(x'/x)}{(y'/y)} = 1 \Rightarrow y' = \left(\frac{x'}{x}\right)y \quad (12)$$

The characteristic parameters for billow (Eq. 9), stress (Eq. 10) and period (Eq. 11) define the behavior of scaled models of the sail according to the relations of the form,

$$w' = w_c w, \quad \text{etc.} \quad (13)$$

with w' the prototype value, w_c the transformation coefficients and w the non-dimensional result.

The boundary conditions on the sail material (Eq. 3) are non-dimensionalized following the same procedure as above to lead to the additional criteria for the prototype boundary loads,

$$F_{yy} = \frac{N_{xc}}{\sqrt[3]{P_1 E h q^2 x_c^2}} = \frac{N_{xc} P_1^{-1/3} x_c^{-2/3}}{\sqrt[3]{E h q^2}} = P_2 \left| (N_x)_{boundary} \right| \quad (14)$$

$$F_{xx} = N_{yc} \sqrt[3]{\frac{P_1^2}{E h q^2 x_c^2}} = \frac{N_{yc} P_1^{2/3} x_c^{-2/3}}{\sqrt[3]{E h q^2}} = P_3 \left| (N_y)_{boundary} \right| \quad (15)$$

$$F_{xy} = N_{xyc} \sqrt[3]{\frac{P_1^{1/2}}{E h q^2 x_c^2}} = \frac{N_{xyc} P_1^{1/6} x_c^{-2/3}}{\sqrt[3]{E h q^2}} = P_4 \left| (N_{xy})_{boundary} \right|. \quad (16)$$

For a case in which an initial prestress is prescribed in the sail, the transformation coefficients N_{ic} provide additional dimensionless parameters. For the case of no prestress (sail stress arises from q only), this can be reduced to unity by proper selection of N_c , here called a characteristic term. The mapping from boundary stresses N_i (stress per sail-thickness) to boom attachment and loading is design specific. The two designs of primary consideration are the ABLE sail design which provides a halyard loading at the corner of the sail and the L'Garde design which distributes the load along the sail border. These specific cases will be considered in a later section. First, the sail model criteria are summarized.

B. Summary of Sail Material Criteria

Table I summarizes the similarity criteria based on the sail material model.

Table I: Summary of Sail Material Model Criteria

| Criteria | Characteristic terms |
|--|---|
| $P_1 = \frac{x_c^2}{y_c^2}$ | $w_c = \sqrt[3]{\frac{q x_c^4}{h E P_1^2}}$ |
| $P_2 = N_{xc} \sqrt[3]{E h q^2 x_c^{-2/3}}$ | $F_c = \sqrt[3]{\frac{E q^2 x_c^8}{h^2 P_1^2}}$ |
| $P_3 = N_{yc} \sqrt[3]{E h q^2 x_c^{-2/3}}$ | $t_c = \sqrt{\frac{\rho h x_c^4}{q P_1}}$ |
| $P_4 = N_{xyc} \sqrt[3]{E h q^2 x_c^{-2/3}}$ | |

This table demonstrates that there are four similarity criteria that need to be satisfied based on this sail model (left hand column of Table I). These criteria consist of 1) geometric similarity of the sail (P_1) and 2) boundary loading conditions (P_2 through P_4).

C. Boom

The supporting boom (support for the sail membrane) is next considered. The equation of motion for a general, uniform boom that considers both flexural and axial rigidity is given as (see Meirovitch [8] for example)

$$(E I v_{ss})_{ss} - (P v_s)_s - f = m v_{tt} \quad (17)$$

where S is the product of the boom's modulus and area moment of inertia, v is the lateral deflection of the boom, m is the boom's mass per unit length, s is the boom length, P is the axial load and f is the distributed normal load on the boom. The sail may also have a point lateral load at the end of the boom of value V . Non-dimensionalization of the boom equation is performed using the approach demonstrated in [4] to result in;

$$\frac{Sv_c}{s_c^4} v_{ssss} - \frac{Pv_c}{s_c^2} v_{ss} - f = \frac{mv_c}{t_c^2} v_{tt} \quad (18)$$

where again the transformation coordinates, v_c , s_c , and t_c are defined as ratios for the corresponding quantities in the model (v , s and t) and in the prototype (v' , s' and t'). This equation is reduced further by dividing through by the coefficient of the first term to yield,

$$v_{ssss} - P_{b1} v_{ss} - 1 = P_{b2} v_{tt} \quad (19)$$

where P_{b1} defines a geometric similarity criteria on the boom (Euler buckling rule);

$$P_{b1} = \frac{Ps_c^2}{S} \quad (20)$$

and dynamic scaling (frequency) between the sail and boom;

$$P_{b2} = \frac{ms^4}{St_c^2} \quad (21)$$

and a characteristic boom displacement due to distributed uniform lateral load is defined as,

$$v_{c,1} = \frac{fs_c^4}{S} \quad (22)$$

The characteristic parameter for boom displacement due to distributed load (Eq. 22) defines the behavior of scaled models of the boom in a manner similar to Eq. 13. The boundary conditions for the boom consist of zero slope and deflection at the boom base, zero bending load applied at the boom tip, and a known shear (lateral load) at the boom tip. The non-zero boundary condition is considered here,

$$Sv_{ss} = V \quad (23)$$

where V is the point lateral load. Equation 23 can be described in a non-dimensional manner to result in a characteristic displacement due to the tip lateral load ($v_{c,2}$),

$$v_{c,2} = \frac{Vs_c^3}{S} \quad (24)$$

D. Summary of Boom Model Criteria

Table II summarizes the similarity criteria that result from the boom model.

Table II: Summary of Boom Model Criteria

| Criteria | Characteristic terms |
|--------------------------------|------------------------------|
| $P_{b1} = \frac{Ps_c^2}{S}$ | $v_{c,1} = \frac{fs_c^4}{S}$ |
| $P_{b2} = \frac{ms^4}{St_c^2}$ | $v_{c,2} = \frac{Vs_c^3}{S}$ |

E. Combining Sail and Boom Models

The boom and sail parameters are interconnected by design resulting in the following additional set of approximate criteria for the system,

$$s_c \propto \frac{x_c}{\sqrt{2}} \quad (25)$$

$$P \propto 2T \propto 2N_c x_c \quad (26)$$

$$V_c \propto q_c \frac{x_c^2 P_1^{-1/2}}{2} \quad (27)$$

$$f \propto q_c \frac{x_c P_1^{-1/2}}{2} \quad (28)$$

representing approximate relationships in length, and forces between the boom and sail.

F. Details associated with Specific Sail Designs:

F.1. ABLE Sail Design:

The ABLE Sail design connects the boom and sail through a halyard at the distal corners of the sail. The loading caused by the halyard is approximated by sail normal stress with

$$\sigma = \frac{T}{ahx_c} \quad (29)$$

where T is the halyard load and a defines a percentage of the sail metric. Greschik and Mikulas [9] present this relationship for sails and give a value $a = .383$. A mapping from halyard loading to Cartesian stress state can be defined as a function of the halyard angle. If the halyard angle is considered fixed, then the mapping value can be absorbed into the parameter a and three boundary stress functions can be described with the proportionality,

$$N_{xc}, N_{yc}, N_{xyc} \propto \frac{T}{x_c} = N_c \quad (30)$$

which defines an equality between the three similarity criteria, P_2 through P_4 . In the case when an initial prestress is prescribed in the sail through an initial halyard load, the transformation coefficient N_c serves as an additional dimensionless parameter that can be used to meet the similarity criteria P_2 through P_4 . The proportionality in Eq. 30 will relate these boundary conditions to halyard loads. Additionally, the point provided by the halyard will result in a tip load on the boom. The characteristic term v_{c1} best approximates the resulting lateral deflection in this case.

F.2. L'Garde Sail Design:

The L'Garde Sail Design prescribes a sail boundary loading at a series of discrete locations along the sail. If the distributed force loads are directed along in a constant direction (parallel to the sail strips), then a similar proportionality exists between the boundary stress functions and distributed load, Q as

$$N_{xc}, N_{yc}, N_{xyc} \propto Q = N_c. \quad (31)$$

If the initial state of the state of the sail is one of zero stress (all sail stress results from pressure loading), then the value N_c provides a characteristic term defining sail loading. The approximate distributed connections between the sail and boom result in a distributed load on the boom. The characteristic term v_{c2} , best approximates the resulting lateral deflection in this case.

G. Summary of Sail System Model Criteria

Table III summarizes the similarity criteria that result from the sail system model.

Table III: Summary of Sail Model Criteria

| Criteria | Characteristic terms |
|--|--|
| $P_1 = \frac{x_c^2}{y_c^2}$ | $w_c = \sqrt[3]{\frac{qx_c^4}{hEP_1^2}}$ |
| $P_2 = N_{xc} \sqrt[3]{Ehq^2 x_c^{-2/3}}$ | $F_c = \sqrt[3]{\frac{Eq^2 x_c^8}{h^2 P_1^2}}$ |
| $P_3 = N_{yc} \sqrt[3]{Ehq^2 x_c^{-2/3}}$ | $t_c = \sqrt{\frac{\rho h x_c^4}{q P_1}}$ |
| $P_4 = N_{xyc} \sqrt[3]{Ehq^2 x_c^{-2/3}}$ | |
| $P_{b1} = \frac{Px_c^2}{2S}$ | $v_{c,1} = \frac{qx_c^5 P_1^{-1/2}}{8S}$ |
| $P_{b2} = \frac{mq}{4S\rho h}$ | $v_{c,2} = \frac{qx_c^5 P_1^{-1/2}}{4\sqrt{2}S}$ |

¹ Terms exists as a similarity criteria if a prestress is defined in the sail, otherwise results in a characteristic definition of boundary loads

² Boom deflection due to tip load characteristic of ABLE Sail design

³ Boom deflection due to distributed lateral load characteristic of L'Garde Sail design

V. Results and Conclusions

A set of similarity criteria are developed for a sail system model based on a thin plate model for the sail that assumes bending stiffness is negligible with respect to extensional stiffness (Von Karmen plate or Foppl membrane equations), a simple uniform boom model that considers both axial and flexural rigidity and corresponding boundary conditions. This set of six similarity criteria define the scaling rules that need to be met in order to maintain similarity in the design of various size, replicating sails (for example test or demonstration sails). These criteria include anticipated parameters such as geometric scaling criteria (P_1) or Euler buckling criteria (P_{b1}) as well as criteria on boundary conditions (P_2 through P_4) and dynamic similarity between the boom and sail (P_{b2}). These criteria are used to define sail system parameters of similar sails as functions of sail size. Figures 1 and 2 demonstrate this process. First, properties associated with the similarity criteria of the sail (P_1 through P_4) are plotted over a range of sail sizes as shown in Fig. 1. Curve P_1 on this figure shows the linear geometric scaling rule, while the exponential scaling of the product of boundary line stresses and solar pressure are shown on curves P_2 through P_4 . Each curve demonstrates constant similarity criteria (P_1 through P_4) for an initial set of these criteria (P_1 selected based on a square sail and P_2 selected based on an initial sail prestress of approximately 1 psi and halyard angle of 26 degrees). Curves P_2 through P_4 can be used for example to determine the sail boundary line stresses (stress per thickness) if a solar pressure is defined. Figure 2 considers the similarity criteria associated with the boom (P_{b1} , P_{b2}). Curves P_{b1} give the required boom stiffness based on invariance of P_{b1} for multiple sail missions. Curves P_{b2} similarly considers the boom stiffness based on dynamic similarity (and assumed boom mass). This plot demonstrates the potential difficulty in defining exact similarity based on the presented system models.

In addition to the similarity criteria, five characteristic terms are defined in the non-dimensionalization process. These terms are not unique since an alternative set of characteristic terms could be defined in the non-dimensionalization process. However, these terms define key elements within the sail system and demonstrate their relation to sail size and orbit. These characteristic terms include sail deflection and stress, w_c , F_c , system period, t_c

and boom deflection for two different loading conditions, v_{c1} , v_{c2} . The exponential relationship to sail size in each variable is noted. Such relationships become important considerations for large sails. Figure 3 shows the characteristic sail deflection and stress (left and right y-axes respectively) as a function of sail size over a range of sail pressure loads (corresponding to 0.25, 0.945, 0.98, 1 AU). This plot demonstrates the exponential relationship between sail size and sail deflection ($x^{4/3}$ for deflection, $x^{8/3}$ for stress). Figure 4 shows the characteristic sail period over a range of sail sizes and solar pressures. Note that sail dynamic response drops with larger sail sails and lower solar pressures. Finally, Fig. 5 demonstrates the characteristic deflections for the boom over a range of sail sizes and solar pressures for both a tip load and uniform lateral load. This series of tables demonstrates the response of characteristic functions to sail scale as well as possible missions. The magnitudes of the characteristic terms, multiplied by the response of the non-dimensional or unit sail system (Eq. 13), is required to get the actual sail response.

Consider now the process of designing a scaled sail system that will accurately characterize (be similar to) a larger future sail system. Based on the model presented here, six criteria are defined as requirements to maintain similarity for a sail system. Eleven unique variables exist within these criteria, (x_c , y_c , E , h , q , m , N_{xc} , N_{yc} , N_{xyc} , S and ρ). Of these variables, x_c will be used to define the scaled sail size while q will be considered a parameter of the sail mission. If the sail material is assumed to be invariant to the scaling process (E , h , ρ), then six parameters remain (y_c , N_{xc} , N_{yc} , N_{xyc} , S and m), the exact number necessary to satisfy the six similarity criteria. However, it is easy to envision additional restrictions placed on the remaining criteria, for example, a functional relationship between boom stiffness and mass (S , m). In such a situation, not all criteria will be met, yielding a non-similar model between the scaled and full-size sail systems. The magnitude of the variance in response will depend on the degree of scale between the two as well as a large number of other factors. Figure 6 considers the variance in result if one of the boundary similarity criteria (P_2 through P_4) is not met (boundary loading between the sail and boom); a linear boundary load rule is applied rather than that specified by criteria P_2 through P_4 . This figure demonstrates that if the departure in loading must remain within one order of magnitude, then the scale factor of up to approximately 3 (2.9) satisfies this.

One of the critical parameters for large sail design is the relation between sail size and resulting boom loads. Based on the similarity criteria P_2 through P_4 and the boundary condition discussion (Eqs. 14-16, ref. 9), the resulting boom load caused by a halyard force T is proportional to sail size, $x_c^{5/3}$ and sail pressure $q^{2/3}$. This relation is generated either from the case in which the boundary conditions are used to satisfy similarity in the sail model (prestress defined in the sail) or when the boundary conditions result as characteristic terms (initial unstressed sail) and assumes non-fixed sail boundaries. This relationship is plotted for a range of sail sizes and solar pressures in Fig. 7. Other relations have been proposed in the literature, included a linear scaling rule (one that could result if the sail boundaries were fixed) or a quadratic scaling rule as demonstrated by Zieders [10]. While a comparison of the assumptions in derivations of these various forms has not yet been performed, each demonstrates the issues in design of large sail systems.

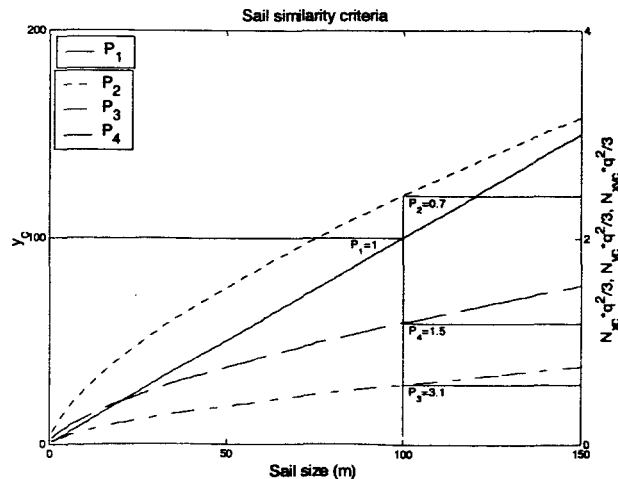


Figure 1: Similarity criteria for the sail

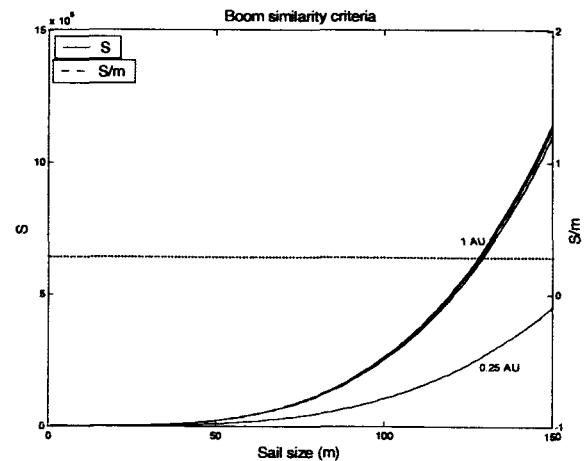


Figure 2: Similarity criteria for the Boom

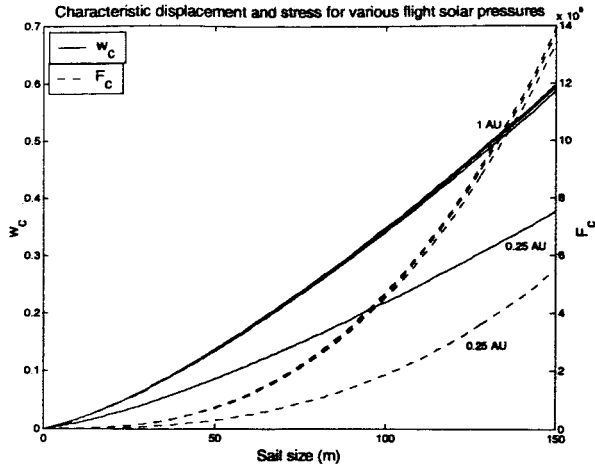


Figure 3: Characteristic displacement and stress

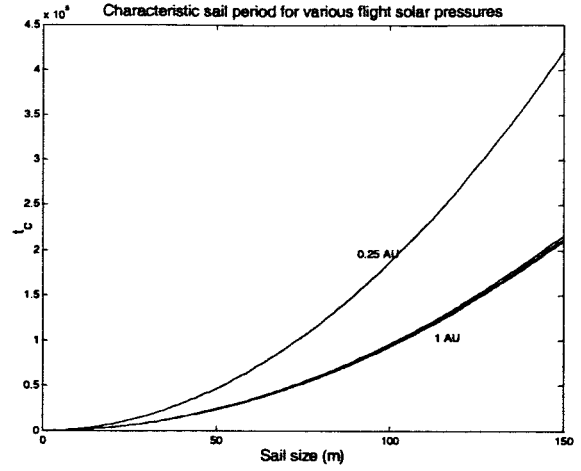


Figure 4: Characteristic period

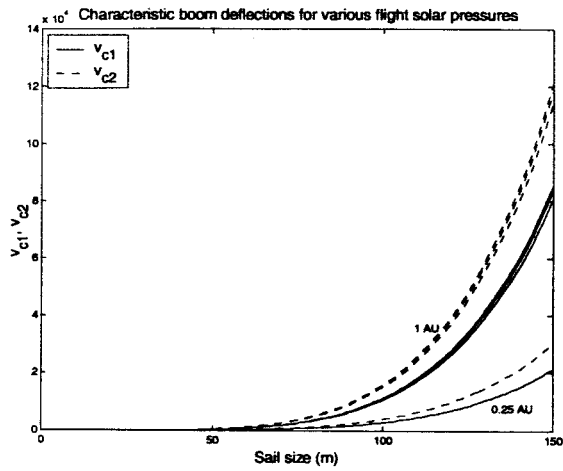


Figure 5: Characteristic boom deflections

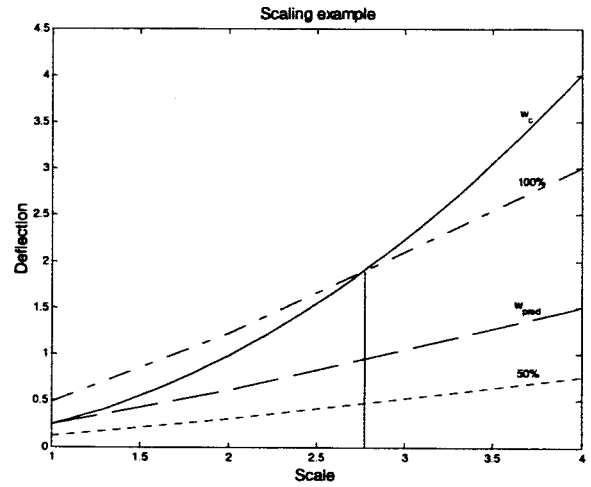


Figure 6: Example of scaling procedure

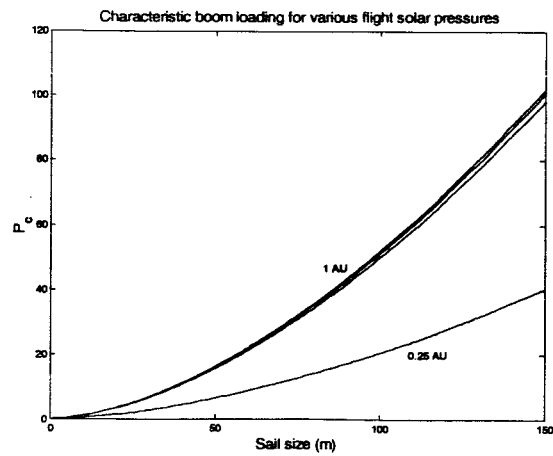


Figure 7: Characteristic boom loading

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